

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

APPLICATION OF NUMERICAL OPTIMIZATION  
TO DESIGN OF COMPOSITE SHAFTS

by

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Application of Numerical Optimization  
to Design of Composite Shafts

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## ABSTRACT

Torsional shafts made from fibrous composite materials are designed for minimum weight. Multiple loading conditions are considered. Design variables are the shaft inner diameter, the ply thicknesses, and the ply orientations. Design constraints include limits on ply strain, displacement, frequency, and Euler and shell buckling. The design task is solved as a numerical optimization problem. Examples are presented to demonstrate the method.



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## I. INTRODUCTION

Optimization and composite materials are two fields that have seen considerable developments in recent years. Advancements in these technologies have provided the design engineer with new dimensions, the depths of which are yet to be explored.

The major material characteristics that are advantageously cultivated in composites are the high strength and stiffness to weight ratios. The weight reduction is so significant that the aircraft and aerospace industries have concentrated major research efforts in this field. Investigations in improving the resistance of composites to environmental factors such as temperature, corrosion, and wear are continuing and findings affirm the contention that the future of composites is bright. However, other industries are slow in utilizing these materials in basic machinery elements, mainly due to a major drawback; the high cost of manufacturing and processing. It is in this vein that the importance of an optimized design becomes apparent. Advancement in fabrication and processing of composite materials coupled with the capability to optimize designs may well lead composites into a more competitive market.

The application of numerical optimization techniques to the design of composite shafts is demonstrated here. The effects of a small mass imbalance on the deflection of a composite shaft with synchronous whirl are included.



Section 2 defines the scope and limitations of the analysis and optimization.

Section 3 describes the analysis of composite shafts, including:

- A. Composite properties
- B. Shaft section properties
- C. Composite failure criteria, ply strain calculations
- D. Displacement and vibration analysis
- E. Summary of failure modes considered

Section 4 presents the optimization results [Refs. 1, 2]. Three design examples are shown.

Section 5 contains the conclusions drawn from this study and recommendations for future investigations.

Appendix A contains a description of the analysis portion of the FORTRAN program used for composite drive shaft design.



## II. SCOPE AND LIMITATIONS

The analysis presented here applies to the shafts made of isotropic or composite materials with pinned-pinned end conditions. Basic shaft design formulas which include the effects of a small mass imbalance are used. The shaft is assumed to rotate with synchronous whirl.

The optimization is constrained to subcritical speeds only. Supercritical speeds introduce feasible design regions that are disjoint from the primary design space. In this investigation, these secondary areas are considered infeasible.

Since the optimization tends to a large radius-small thickness design, radial stresses are neglected. However, buckling as a thin cylinder due to torsion or compression is considered. The equations used for these failure criteria do not include dynamic effects. A constant torque, centrifugal force, and axial force are considered in the formulas [Ref. 3].

The analysis incorporates the ability to design a shaft that can be used in two or more loading conditions. For example, a shaft may be designed to transmit 50 HP at 300 RPM with an axial load of 2000 lbs. as well as to transmit 150 HP at 3000 RPM with a 1000 lbs. axial load.





### III. ANALYSIS

The problem considered here is a shaft required to transmit a specified horsepower at a given speed. Basic equations are used throughout the analysis.

The inch-pound-second system of units (IPS) is used in the computer program analysis. However, by changing the equation for torque and the acceleration due to gravity to reflect the International System of Units, the program may be used with consistent SI units.

The assumptions used in the analysis are:

1. The plies are made of orthotropic material, but need not all be the same material.
2. Structure:
  - a. Plane stress in radial direction
  - b. Plies are at different orientations and have different thicknesses.
  - c. Cross-section is constant over entire length
3. Loads: (see Figure 1)
  - a. Axial force
  - b. Constant torque
  - c. Centrifugal force
4. Failure modes:

Typical ply orientation and composite layup are shown in Figure 2.



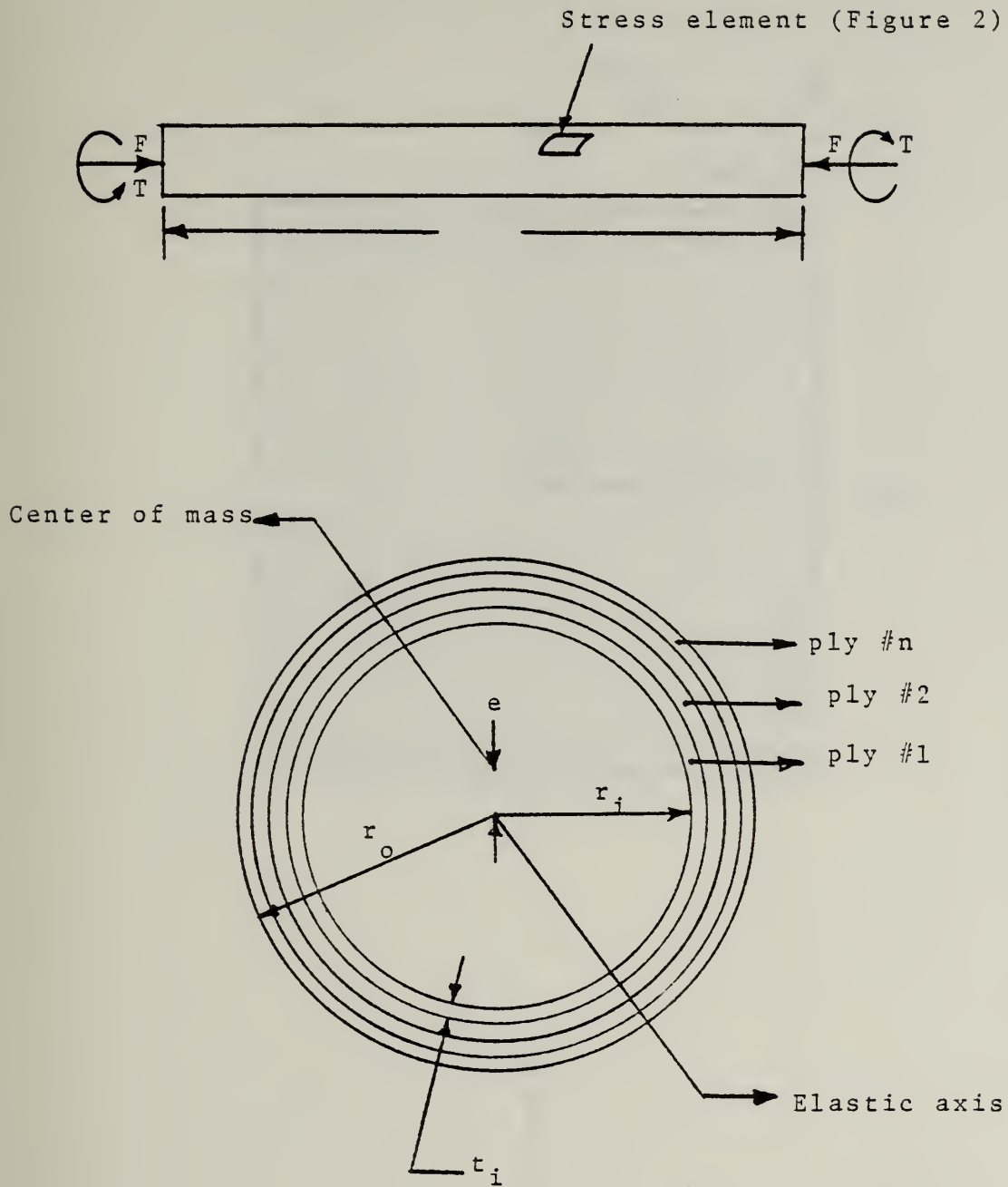


Figure 1. Applied Loads: Axial load ( $F$ ), Mass imbalance ( $e$ ), Torque ( $T$ ).



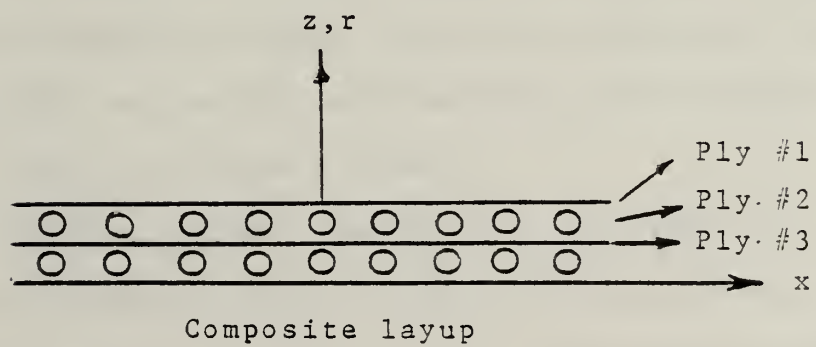
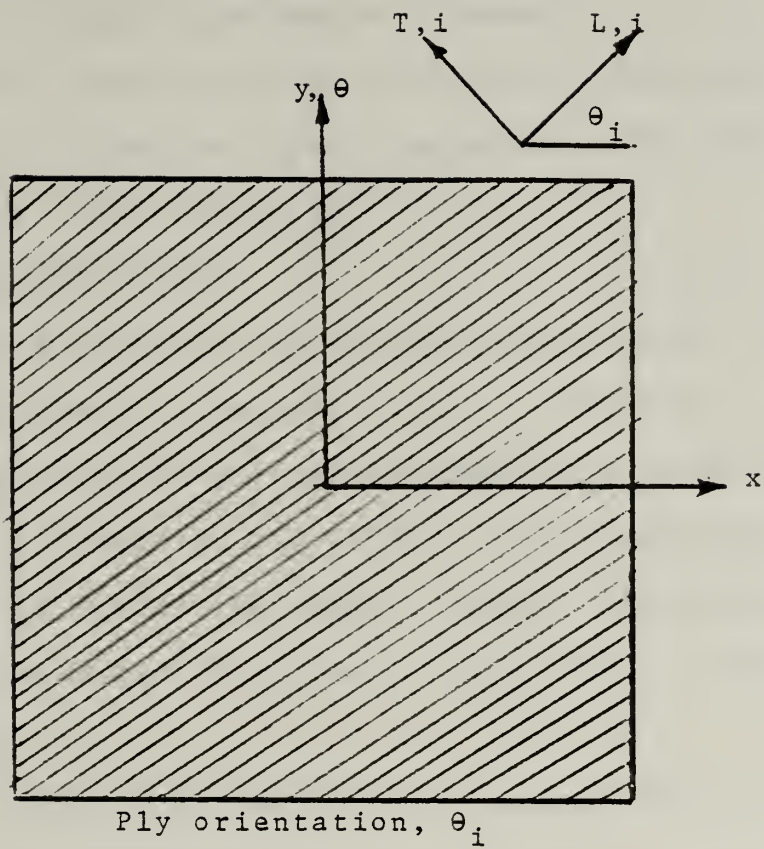


Figure 2. Typical ply orientation and composite layup.



- a. Maximum ply strain limits
- b. Maximum displacement
- c. First fundamental frequency
- d. Shell buckling (axial and torsional)
- e. Euler buckling (contained within the displacement constraint)

#### A. COMPOSITE PROPERTIES

Stress is a measure of internal force within a body. This together with elastic constants are the variables for the determination of stiffness and strength of a material. The mechanism of deformation and failure are interpreted in terms of the state of stress and strain. When discussing stress it is usually considered to be the average stress over some physical dimension.

In Composites, three levels of average stress are considered:

- a. Micromechanical or local stress is that calculation based on distinct, continuous phases of fiber, matrix and, in some cases, the interphase and voids.
- b. Ply stress is that calculation based on assumed homogeneity within each ply or ply group where the fiber and matrix are smeared and no longer recognized as distinct phases.
- c. Laminate stress resultant  $N$  or moment  $M$  is based on an average of ply stresses across the thickness of a laminate. The individual plies are smeared.





The coefficients, or material constants, of the stress-strain relations can be packaged in set of engineering constants, compliance components or modulus components.

- a. MODULUS is used to calculate the stress from strain. This is the basic set needed for the stiffness of multidirectional laminates.
- b. COMPLIANCE is used to calculate the strain from stress. This is the set needed for the calculation of engineering constants.
- c. ENGINEERING CONSTANTS are the carryover from the conventional materials. Designers often feel more comfortable working with the familiar engineering constants.
- d. There is a direct relationship between the modulus and compliance. One is the inverse of the other.

The stiffness of unidirectional composites is governed by the same stress-strain relation that is valid for isotropic materials. However the number of independent constants are four for planar composites as compared to two for isotropic materials. The stiffness of isotropic materials can be represented by the Young's modulus alone because the Poisson's ratio is the same all directions.

For composite materials, Poisson's ratios are not bounded and can have very significant effect. Young's modulus alone is not sufficient to describe the stiffness of composite materials. It is not sufficient for isotropic materials either.



The ply stiffness related to the global coordinate system are [Ref. 4]:

$$Q_{11} = m^4 Q_{xx} + n^4 Q_{yy} + 2m^2 n^2 Q_{xy} + 4m^2 n^2 Q_{ss}$$

$$Q_{22} = n^4 Q_{xx} + m^4 Q_{yy} + 2m^2 n^2 Q_{xy} + 4m^2 n^2 Q_{ss}$$

$$Q_{12} = m^2 n^2 Q_{xx} + m^2 n^2 Q_{yy} + (m^4 + n^4) Q_{xy} - 4m^2 n^2 Q_{ss}$$

$$Q_{66} = m^2 n^2 Q_{xx} + m^2 n^2 Q_{yy} - 2m^2 n^2 Q_{xy} + (m^2 - n^2) Q_{ss}$$

$$Q_{16} = m^3 n Q_{xx} - m n^3 Q_{yy} + (m n^3 - m^3 n) Q_{xy} + 2(m n^3 - m^3 n) Q_{ss}$$

$$Q_{26} = m n^3 Q_{xx} - m^3 n Q_{yy} + (m n^3 - m n^3) Q_{xy} + 2(m^3 n - m n^3) Q_{ss}$$

where

$$m = \cos \theta$$

$$n = \sin \theta$$

$$Q_{xx} = \frac{E_x}{1 - \nu_x \nu_y}$$

$$Q_{yy} = \frac{E_y}{1 - \nu_x \nu_y}$$

$$Q_{xy} = \frac{\nu_y E_x}{1 - \nu_x \nu_y}$$

$$Q_{ss} = E_s$$

and  $Q_{xx}$ ,  $Q_{yy}$ ,  $Q_{xy}$ ,  $Q_{ss}$ : On-axis modulus components referred to the local coordinate system

$Q_{iJ}$ : Off-axis modulus components;  $i, J = 1, 2, 6$

## B. SHAFT SECTION PROPERTIES

Axial stiffness, bending stiffness, and torsional stiffness are smeared properties.



a. Axial stiffness (AE)

$$AE = \sum_{k=1}^{NPLY} (Q_{11})_k A_k \quad (3B.1)$$

b. Bending stiffness (EI)

$$EI = \sum_{k=1}^{NPLY} (Q_{11})_k I_k \quad (3B.2)$$

c. Torsional stiffness (GJ)

$$GJ = \sum_{k=1}^{NPLY} (Q_{11})_k J_k \quad (3B.3)$$

where

$Q_{ij}$  : the component of modulus ;  $i, j = 1, 2, 6$

$I_k$  : the ply moment of inertia ;  $k = 1, NPLY$

$J_k$  : the ply polar moment of inertia ;  $k = 1, NPLY$

$A_k$  : the ply cross-sectional area ;  $k = 1, NPLY$

NPLY : the number of composite plies.

## C. COMPOSITE FAILURE CRITERIA

For the determination of strength of any material it is usual practice to estimate the stress at the time and location when failure occurs. In the case of conventional materials it is needed only to determine the maximum tensile, compressive, or shear stress or the principal stresses and then some observation can be made about the failure mechanism. This process is relatively straightforward because isotropic materials have



no preferential orientation and usually one strength constant will suffice. The isotropic material is essentially a one-dimensional or one constant material. The Young's modulus for stiffness will suffice because Poisson's ratio is the same in all directions, and uniaxial tensile strength will also suffice because the shear strength is taken to be about 50 to 60 percent of the tensile strength.

For composite materials, however, the on-constant approach for stiffness or for strength is no longer adequate. Four elastic constants are needed for the stiffness of a planer element. Six constants for the strength of unidirectional composites are needed. Unidirectional composites have directionally dependent strengths. The longitudinal strength can be twenty times that of the transverse and shear strengths; so for any state of stress, all three stress components must be examined before a judgement on the cause of failure can be made. The specific stress component that is responsible for the failure may not be easily identified. Probably all three components are responsible. The effect of combined stresses must be systematically determined and can be regarded as a way of life for composites.

For composite materials, there is needed a failure criterion for the unidirectional plies. The strength of a laminated composite will be based on the strength of the individual plies within a laminate. Successive ply failures are expected as the applied load to a laminate increase. The first ply failure is followed by other ply failures until the last ply failure





which would be the ultimate failure of the laminate. The ply stress and ply strain calculations are intended for strength determination. There are two popular approaches for failure criteria of unidirectional composites. They are each based on the on-axis stress or strain as the basic variable [Ref. 4].

1. The Maximum Stress Criteria

$$\sigma_x \leq \bar{\sigma}_x$$

$$\sigma_y \leq \bar{\sigma}_y$$

$$\sigma_s \leq \bar{\sigma}_s$$

Failure occurs when one of the qualities is met.

2. The Maximum Strain Criteria

$$\epsilon_x \leq \bar{\epsilon}_x$$

$$\epsilon_y \leq \bar{\epsilon}_y$$

$$\epsilon_s \leq \bar{\epsilon}_s$$

Failure occurs when one of the equalities is met.

The maximum stress and strain criteria are not the same. Only when Poisson's ratio of the unidirectional materials is zero, the criteria become identical. Conceptually they are similar. Each component of stress or strain has its own criterion and is not affected by the other components. There is assumed to be no interaction.

In this study the ply strain failure criteria was used.

#### D. DISPLACEMENT AND VIBRATION ANALYSIS

In order to do the displacement and vibration analysis the deflected shape of beam is assumed as:



$$\delta = \delta_0 \sin \frac{\pi x}{\ell} \quad (\text{see Figure 3})$$

where the beam is assumed to be simply supported. The deflection is maximum at  $x = 1/2$  [Ref. 3]

$$\delta_{\max} = \frac{5\ell^4}{384} \left[ \frac{K_1 e + K_2}{EI - K_1 \left(\frac{\ell}{\pi}\right)^4 - F \left(\frac{\ell}{\pi}\right)^2} \right] \quad (3D.1)$$

where:

$$K_1 = \rho A \omega^2$$

$$K_2 = \rho A g$$

$\rho$  = mass per unit volume

$A$  = cross sectional area

$\omega$  = shaft speed in radians per second

$g$  = acceleration due to gravity

$e$  = eccentricity of mass with respect to the axis of rotation

$\ell$  = shaft length

$EI$  = smeared bending stiffness

$F$  = axial load

Certain characteristics of this equation are of particular interest. If there is no rotation or axial load, the formula reduces to the classical equation for maximum beam deflection under its own weight.



$$\delta_{\max} = \frac{5K_2 l^4}{384EI} \quad (3D.2)$$

If there is no axial load, the equation becomes unstable (deflection goes infinity) when  $EI = K_1 \left(\frac{l}{\pi}\right)^4$  or, solving for the critical speed,

$$\omega_c = \pi^2 \sqrt{\frac{EI}{\rho A l^4}} \quad (3D.3)$$

which defines the first fundamental frequency of a simply supported shaft.

If there is no rotation, instability arises when

$$F_c = \frac{\pi^2 EI}{l^2} \quad (3D.4)$$

recognizable as Euler's column buckling criteria.

With both rotation and axial load, the critical speed becomes

$$\omega_c = \pi^2 \sqrt{\frac{EI - F \left(\frac{l}{\pi}\right)^2}{\rho A l^4}} \quad (3D.5)$$

To avoid the instability regions, the denominator of equation (3D.1) must be given a lower bound greater than zero (point A in Figure 4). Additionally, a maximum deflection has to be imposed to prevent computer overflow (point B in Figure 4).



These limits have no relevance with the deflection constraint used in the design optimization process (point C in Figure 4), but are used only to prevent numerical ill-conditioning on the computer.

#### E. SUMMARY OF FAILURE MODES CONSIDERED

Failure criteria serve important functions in the design and sizing of composite laminates. They should provide a convenient framework or model for mathematical operations. The framework should remain the same for different definitions of failures, such as the ultimate strength, the proportional limit, yielding, endurance limit, or a working stress based on design or reliability considerations. Failures in composite materials involve many modes, such as fiber failures, matrix failures, interfacial failures, delamination, and buckling. Furthermore, the various modes interact and can occur concurrently or sequentially.

NLC(6(NPLY)+4) modes of failure are considered in this investigation. The 6(NPLY)(NLC) give the maximum strain limits in longitudinal compressive strain, longitudinal tensile strain, transverse compressive strain, transverse tensile strain, negative shear strain, and positive shear strain. Failure is assumed if a ply strain multiplied by a safety factor is greater than the corresponding strain limit.

A limit on the deflection is imposed. A common practice is to specify a maximum deflection in inches per foot of shaft





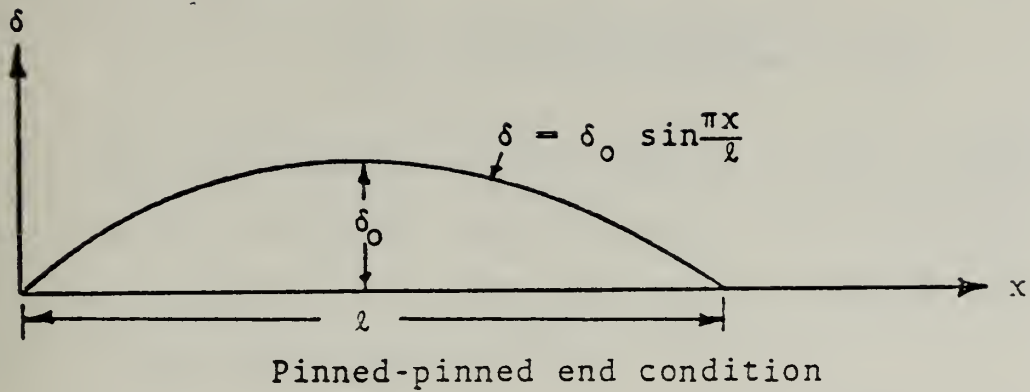


Figure 3. Assumed deflected shape of the shaft.

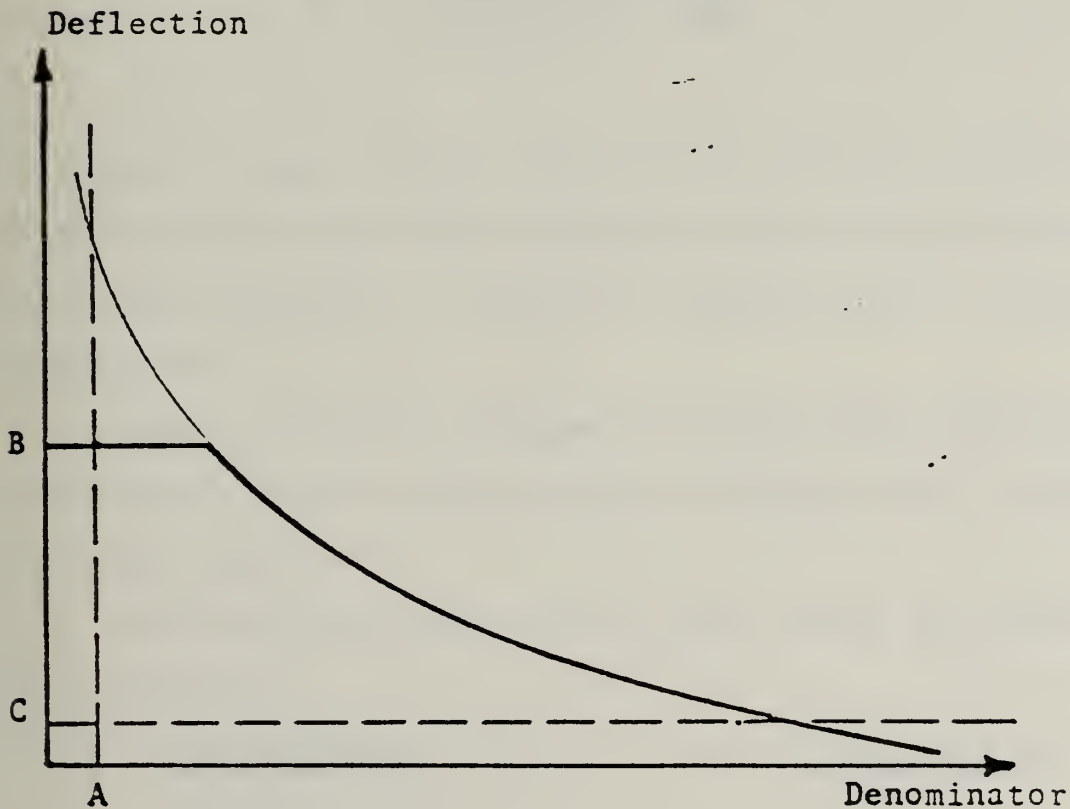


Figure 4. Limits on the deflection. The solid curve represents the deflection values used in the analysis.



length. For example, as used in this report,

$$\delta_{\max} = 0.005 \text{ inch/foot length}$$

Since the optimization may lead to a thin cylinder design, buckling due to torsion and compression are considered. The critical torque and critical compressive stress for buckling of thin cylinders are [Refs. 5, 6]:

$$T_c = \frac{\pi \sqrt{2} E}{3(1-\nu^2)^{0.75}} \sqrt{rt^5} \approx \frac{\pi \sqrt{2} Q_{11}}{3} \sqrt{rt^5}$$

$$\sigma_c = \frac{Et}{r \sqrt{3(1-\nu^2)}} \approx \frac{tQ_{11}}{\sqrt{3}r}$$

Experimental data indicate that actual failure of the cylinders usually occurs below fifty percent of the values calculated from these equations. Therefore a large factor of safety is needed here.

Lastly, failure is assumed if a specified percentage of the critical speed as calculated by equation (3D.5) is less than the shaft speed.

The factors of safety used in this study are listed in the following:

<u>FAILURE MODE</u>	<u>SAFETY FACTOR</u>
Strain	2
Torsion Buckling	5
Shell Buckling	5
Frequency	5
Deflection	1



#### IV. OPTIMIZATION RESULTS

There are a number of optimization programs available, each one using different techniques in locating the desired optimum design. COPES/CONMIN [Refs. 1,2] is a versatile program that may be used for sensitivity analysis as well as an optimization tool. Provided with a user supplied analysis program (Subroutine ANALIZ) where the objective function, constraints and other relevant parameters are calculated, it determines a usable and feasible sector from which a search direction is chosen. This choice of search direction is made by using such information as the gradients of the objective function and gradients of active and violated constraints. This iterative process of minimizing/maximizing the objective function by changing the design variables is terminated when no further improvement can be made.

In this study, the objective function to be minimized is the weight, with the shaft inner diameter and the ply thicknesses and orientations as design variables. The analysis subroutine incorporates the analysis of composite shafts described in the previous sections. Several design examples are shown to demonstrate the effects of the different parameters on the optimized design. For the examples considered here, the loads are listed in Table 1 together with the optimum weight and the constraints are listed in Table 2.



Table 1: Optimization Examples

No.	HP	RPM	Axial Load (lbs)	Material	Optimum Weight (lbs)
1	150	300	0	Steel	50.2
2	150	300	0	G/E	12.5
3	1000	6000	3000	Steel	26.5
	1500	8000	0		
4	1000	6000	3000	G/E	10.8
	1500	8000	0		
5	25000	110	40000	Steel	24822.0
	10000	30	5000		
6	25000	110	40000	G/E	1872.5
	10000	30	5000		

Examples No. 1 and No. 2 are designs for one load condition.

Shaft lengths are 120 inches here.

Examples No. 3 and No. 4 are designs for two load conditions.

Shaft lengths are 60 inches here.

Examples No. 5 and No. 6 are designs for two load conditions.

Shaft lengths are 240 inches here.

The results of optimization are presented in Figures 5-16.





Table 2: Design Constraints

<u>Number</u>	<u>Constraint</u>
For NLC = 1	
1 through 6(NPLY)	Maximum Strain Limits
6(NPLY)+1	Buckling due to Torque
6(NPLY)+2	Buckling due to Compression
6(NPLY)+3	Speed
6(NPLY)+4	Deflection
For NLC = 2	
6(NPLY)+5 through 12(NPLY)+4	Maximum Strain Limits
12(NPLY)+5	Buckling due to Torque
12(NPLY)+6	Buckling due to Compression
12(NPLY)+7	Speed
12(NPLY)+8	Deflection
Etc. for NLC > 2	



COMPOSITE CRIVESHAF OUTPUT

NUMBER OF PLYS = 1  
 NUMBER OF MATERIAL TYPES = 1  
 NUMBER OF LOAD CONDITIONS = 1  
 ECCENTRICITY = 0.10000E+00

DIMENSIONS  
 PLY THICKNESS PERCENT DIAMETER THETA  
 1 0.10000E+01 100.00 0.40000E+01 0.0

STIFFNESS  
 AF = 0.31071E+09  
 EI = 0.36838E+09  
 GJ = 0.27187E+09

LOADS:  
 L.C. 1 T 0.31513E+05 0.0 M 0.0 F 0.15000E+03 H.P. 0.20000E+03 RPM

LOAD CONDITION 1  
 PLY 1 0.0 EFL 100.00 S.F. 0.0 EPT 100.00 S.F. 0.23182E-03 S.F. 5.61  
 CRITICAL SPEED = 0.16278E+03  
 MAXIMUM DEFLECTION = 0.24099E-01  
 WEIGHT = 0.31893E+03  
 VOLUME = 0.11310E+04

Figure 5. Initial Design for Example 1.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 1  
 NUMBER OF MATERIAL TYPES = 1  
 NUMBER OF LOAD CONDITIONS = 1  
 ECCENTRICITY = 0.10000E+00

DIMENSIONS  
 PLY THICKNESS PERCENT DIAMETER THETA  
 1 0.81786E-01 100.00 0.58543E+01 0.0

STIFFNESS  
 AE = 0.48696E+08  
 EI = 0.20370E+09  
 GJ = 0.14259E+09

LOADS:  
 L.C. 1 T 0.31513E+05 0.0 M 0.0 F 0.0 H.P. 0.15000E+03 204 0.30000E+03

LOAD CONDITION 1  
 PLY 0.0 EPL 100.00 S.F. 0.0 SPT 100.00 S.F. 0.0 EPLT 0.54690E-03 S.F. 2.01  
 CRITICAL SPEED = 0.29717E+03  
 MAXIMUM DEFLECTION = 0.70401E-02  
 WEIGHT = 0.50191E+02  
 VOLUME = 0.17798E+03

Figure 6. Optimum Design for Example 1.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4  
NUMBER OF MATERIAL TYPES = 1  
NUMBER OF LOAD CONDITIONS = 1  
ECCENTRICITY = 0.10000E+00

DIMENSIONS  
PLY THICKNESS PERCENT DIAMETER THETA  
INSIDE 0.20000E+01  
1 0.25000E+00 25.00 0.25000E+01 0.0  
2 0.25000E+00 25.00 0.30000E+01 0.20000E+02  
3 0.25000E+00 25.00 0.35000E+01 -0.20000E+02  
4 0.25000E+00 25.00 0.40000E+01 0.90000E+02

STIFFNESS  
AE = 0.12143E+09  
EI = 0.12426E+09  
GJ = 0.37225E+08

LOADS:  
L.C. T M F H.P. GPM  
1 0.31513E+05 0.0 0.0 0.15000E+03 0.30000E+03

LOAD CONDITION  
PLY EPL S.F. EPT S.F. EPLT S.F.  
1 0.0 100.00 0.0 100.00 0.10582E-02 17.39  
2 0.40811E-03 21.00 -0.40811E-03 43.13 0.97273E-03 18.62  
3 -0.47613E-03 18.00 0.47613E-03 9.89 0.11348E-02 16.21  
4 -0.53159E-09 100.00 -0.53159E-09 100.00 -0.16931E-02 10.87  
CRITICAL SPEED = 0.20662E+03  
MAXIMUM DEFLECTION = 0.14741E-01  
WEIGHT = 0.63334E+02  
VOLUME = 0.11310E+04

Figure 7. Initial Design for Example 2.





COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4  
 NUMBER OF MATERIAL TYPES = 1  
 NUMBER OF LOAD CONDITIONS = 1  
 ECCENTRICITY = 0.10000E+00

PLY	THICKNESS	PERCENT	DIAMETER	THETA
	INSIDE	0.12806E+01		
1	0.14901E-07	0.00	0.12806E+01	0.0
2	0.14072E+00	50.00	0.16421E+01	0.32225E+02
3	0.14072E+00	50.00	0.20035E+01	-0.32225E+02
4	0.14901E-07	0.00	0.20035E+01	0.90000E+02

STIFFNESS  
 AF = 0.21642E+08  
 EI = 0.76481E+07  
 GJ = 0.60767E+07

LOADS:  
 L.C. 1 T 0.31513E+05 0.0 M 0.0 F 0.0 H.P. 0.15000E+03 RPM 0.30000E+03

LOAD CONDITION	PLY	EPL	S.F.	EPT	S.F.	EDIT	S.F.
1	0.0	100.00	0.0	100.00	0.33206E-02	5.54	
2	0.19207E-02	4.46	-0.19207E-02	9.16	0.18364E-02	10.02	
3	-0.23435E-02	3.66	0.23435E-02	2.01	0.22406E-02	8.21	
4	0.16311E-08	100.00	-0.16311E-08	100.00	-0.51950E-02	3.54	

CRITICAL SPEED = 0.11524E+03  
 MAXIMUM DEFLECTION = 0.50004E-01  
 WEIGHT = 0.12530E+02  
 VOLUME = 0.22376E+03

Figure 8. Optimum Design for Example 2.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 1  
NUMBER OF MATERIAL TYPES = 1  
NUMBER OF LOAD CONDITIONS = 2  
ECCENTRICITY = 0.10000E+00

DIMENSIONS  
PLY THICKNESS PERCENT DIAMETER THETA  
INSIDE 0.10000E+02  
1 0.10000E+01 100.00 0.12000E+02 0.0

STIFFNESS  
AF = 0.11393E+10  
EI = 0.17374E+11  
GJ = 0.12162E+11

LOADS:  
L.C. T M F H.P. RPM  
1 0.10504E+05 0.0 0.30000E+04 0.10000E+04 0.60000E+04  
2 0.11817E+05 0.0 0.0 0.15000E+04 0.30000E+04

LOAD CONDITION 1  
PLY EFL S.F. EPT S.F. EPLT S.F.  
1 0.26333E-05 100.00 0.0 100.00 0.51823E-05 100.00  
CRITICAL SPEED = 0.22742E+04  
MAXIMUM DEFLECTION = 0.10584E-01

LOAD CONDITION 2  
PLY EFL S.F. EPT S.F. EPLT S.F.  
1 0.0 100.00 0.0 100.00 0.58301E-05 100.00  
CRITICAL SPEED = 0.22743E+04  
MAXIMUM DEFLECTION = 0.20022E-01  
WEIGHT = 0.58471E+03  
VOLUME = 0.20735E+04

Figure 9. Initial Design for Example 3.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 1  
 NUMBER OF MATERIAL TYPES = 1  
 NUMBER OF LOAD CONDITIONS = 2  
 ECCENTRICITY = 0.10000E+00

DIMENSIONS  
 PLY THICKNESS PERCENT DIAMETER THETA  
 1 0.49525E-01 100.00 0.10098E+02 0.0

STIFFNESS  
 AE = 0.51542E+08  
 EI = 0.65059E+09  
 GJ = 0.45541E+09

LOADS:  
 L.C. T M F H.P. RPM  
 1 0.10504E+05 0.0 0.30000E+04 0.10000E+04 0.60000E+04  
 2 0.11817E+05 0.0 0.0 0.15000E+04 0.30000E+04

LOAD CONDITION 1  
 PLY EPL S.F. EPT S.F. EPLT S.F.  
 1 0.58205E-04 17.18 0.0 100.00 0.11646E-03 11.16  
 CRITICAL SPEED = 0.20673E+04  
 MAXIMUM DEFLECTION = 0.13034E-01

LOAD CONDITION 2  
 PLY EPL S.F. EPT S.F. EPLT S.F.  
 1 0.0 100.00 0.0 100.00 0.13102E-03 9.92  
 CRITICAL SPEED = 0.20691E+04  
 MAXIMUM DEFLECTION = 0.25007E-01  
 WEIGHT = 0.26454E+02  
 VOLUME = 0.93807E+02

Figure 10. Optimum Design for Example 3.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4  
 NUMBER OF MATERIAL TYPES = 1  
 NUMBER OF LOAD CONDITIONS = 2  
 ECCENTRICITY = 0.10000E+00

DIMENSIONS  
 PLY THICKNESS PERCENT DIAMETER THETA  
 INSIDE 0.60000E+01  
 1 0.25000E+00 25.00 0.65000E+01 0.0  
 2 0.25000E+00 25.00 0.70000E+01 0.20000E+02  
 3 0.25000E+00 25.00 0.75000E+01 -0.20000E+02  
 4 0.25000E+00 25.00 0.80000E+01 0.90000E+02

STIFFNESS  
 AE = 0.29855E+09  
 EI = 0.17211E+10  
 GJ = 0.45088E+09

LOADS:  
 L.C. T M F H.P. RPM  
 1 0.10504E+05 0.0 0.30000E+04 0.10000E+04 0.60000E+04  
 2 0.11817E+05 0.0 0.0 0.15000E+04 0.40000E+04

LOAD CONDITION 1  
 PLY EPL S.F. EPT S.F. EPLT S.F.  
 1 0.10049E-04 100.00 0.0 100.00 0.75716E-04 100.00  
 2 0.35080E-04 100.00 -0.25031E-04 100.00 0.56005E-04 100.00  
 3 -0.19205E-04 100.00 0.29254E-04 100.00 0.73384E-04 100.00  
 4 0.29259E-10 100.00 0.10049E-04 100.00 -0.93189E-04 100.00  
 CRITICAL SPEED = 0.20130E+04  
 MAXIMUM DEFLECTION = 0.13325E-01

LOAD CONDITION 2  
 PLY EPL S.F. EPT S.F. EPLT S.F.  
 1 0.0 100.00 0.0 100.00 0.85181E-04 100.00  
 2 0.29482E-04 100.00 -0.29482E-04 100.00 0.70272E-04 100.00  
 3 -0.31588E-04 100.00 0.31588E-04 100.00 0.75291E-04 100.00  
 4 0.32917E-10 100.00 -0.32917E-10 100.00 -0.10484E-03 100.00  
 CRITICAL SPEED = 0.20136E+04  
 MAXIMUM DEFLECTION = 0.26695E-01  
 WEIGHT = 0.73890E+02  
 VOLUME = 0.13195E+04

Figure 11. Initial Design for Example 4.





# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4  
NUMBER OF MATERIAL TYPES = 1  
NUMBER OF LOAD CONDITIONS = 2  
ECCENTRICITY = 0.10000E+00

DIMENSIONS		PERCENT	DIAMETER	THETA
PLY	THICKNESS			
	INSIDE	0.54431E+01		
1	0.15094E+00	82.87	0.57450E+01	0.0
2	0.15506E-01	8.57	0.57762E+01	0.86400E+01
3	0.15606E-01	8.57	0.58074E+01	-0.86400E+01
4	0.59257E-11	0.00	0.58074E+01	0.90000E+02

STIFFNESS  
AE = 0.67352E+08  
EI = 0.26658E+09  
GJ = 0.18559E+08

LOADS:	T	M	F	H.P.	RPM
L.C. 1	0.10504E+05	0.0	0.30000E+04	0.10000E+04	0.60000E+04
2	0.11817E+05	0.0	0.0	0.15000E+04	0.30000E+04

LOAD CONDITION		1		2	
PLY	FPL	S.F.	EPT	S.F.	FPLT
1	0.44542E-04	100.00	0.0	100.00	0.16223E-02
2	0.28580E-03	29.79	-0.24125E-03	72.95	0.15443E-02
3	-0.20003E-03	42.84	0.24457E-03	19.26	0.15792E-02
4	0.51451E-09	100.00	0.44542E-04	100.00	-0.16400E-02

CRITICAL SPEED = 0.20671E+04  
MAXIMUM DEFLECTION = 0.13038E-01

LOAD CONDITION		2		3	
PLY	FPL	S.F.	EPT	S.F.	FPLT
1	0.0	100.00	0.0	100.00	0.18251E-02
2	0.27254E-03	31.44	-0.27254E-03	64.58	0.17522E-02
3	-0.27402E-03	31.24	0.27402E-03	17.19	0.17617E-02
4	0.57928E-09	100.00	-0.57928E-09	100.00	-0.18450E-02

CRITICAL SPEED = 0.20713E+04  
MAXIMUM DEFLECTION = 0.24942E-01  
WEIGHT = 0.10816E+02  
VOLUME = 0.19314E+03

Figure 12. Optimum Design for Example 4.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 1  
NUMBER OF MATERIAL TYPES = 1  
NUMBER OF LOAD CONDITIONS = 2  
ECCENTRICITY = 0.10000E+00

DIMENSIONS  
PLY THICKNESS PERCENT DIAMETER THETA  
INSIDE 0.50000E+01  
1 0.80000E+01 100.00 0.21000E+02 0.0

STIFFNESS  
AE = 0.10771E+11  
EI = 0.31371E+12  
GJ = 0.21960E+12

LOADS:  
L.C. T M F H.P. RPM  
1 0.14324E+08 0.0 0.40000E+05 0.25000E+05 0.11000E+03  
2 0.21008E+08 0.0 0.50000E+04 0.10000E+05 0.30000E+02

LOAD CONDITION 1  
PLY EPL S.F. EPT S.F. EPLT S.F.  
1 0.37136E-05 100.00 0.0 100.00 0.68490E-03 1.00  
CRITICAL SPEED = 0.19636E+03  
MAXIMUM DEFLECTION = 0.13179E-01

LOAD CONDITION 2  
PLY EPL S.F. EPT S.F. EPLT S.F.  
1 0.46420E-06 100.00 0.0 100.00 0.10045E-02 1.27  
CRITICAL SPEED = 0.19643E+03  
MAXIMUM DEFLECTION = 0.12725E-01  
WEIGHT = 0.22113E+05  
VOLUME = 0.78414E+05

Figure 13. Initial Design for Example 5.



# COMPOSITE CRIVESHAFT OUTPUT

NUMBER OF PLYS = 1  
 NUMBER OF MATERIAL TYPES = 1  
 NUMBER OF LOAD CONDITIONS = 2  
 ECCENTRICITY = 0.10000E+00

DIMENSIONS  
 PLY THICKNESS PERCENT DIAMETER THETA  
 1 0.63589E+01 100.00 0.24718E+02 0.0

STIFFNESS  
 AE = 0.12091E+11  
 EI = 0.57052E+12  
 GJ = 0.39936E+12

LOADS:  
 L.C. T M F H.P. RPM  
 1 0.14324E+08 0.0 0.40000E+05 0.25000E+05 0.11000E+03  
 2 0.21008E+08 0.0 0.50000E+04 0.10000E+05 0.30000E+02

LOAD CONDITION 1  
 PLY PPL S.F. EPT S.F. EPLT S.F.  
 1 0.33083E-05 100.00 0.0 100.00 0.44328E-03 2.93  
 CRITICAL SPEED = 0.24998E+03  
 MAXIMUM DEFLECTION = 0.81212E-02

LOAD CONDITION 2  
 PLY PPL S.F. EPT S.F. EPLT S.F.  
 1 0.41353E-06 100.00 0.0 100.00 0.65014E-03 2.00  
 CRITICAL SPEED = 0.25007E+03  
 MAXIMUM DEFLECTION = 0.73531E-02  
 WEIGHT = 0.24822E+05  
 VOLUME = 0.88022E+05

Figure 14. Optimum Design for Example 5.



# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4  
NUMBER OF MATERIAL TYPES = 1  
NUMBER OF LOAD CONDITIONS = 2  
ECCENTRICITY = 0.10000E+00

DIMENSIONS  
PLY THICKNESS PERCENT DIAMETER THETA  
INSIDE 0.50000E+01  
1 0.20000E+01 25.00 0.70000E+01 0.0  
2 0.20000E+01 25.00 0.13000E+02 0.20000E+02  
3 0.20000E+01 25.00 0.17000E+02 -0.20000E+02  
4 0.20000E+01 25.00 0.21000E+02 0.90000E+02

STIFFNESS  
AE = 0.38749E+10  
EI = 0.78896E+11  
GJ = 0.27562E+11

LOADS:  
L.C. T M F H.P. RPM  
1 0.14324E+08 0.0 0.40000E+05 0.25000E+05 0.11000E+03  
2 0.21000E+08 0.0 0.50000E+04 0.10000E+05 0.30000E+02

LOAD CONDITION 1  
PLY EPL S.F. EPT S.F. EPLT S.F.  
1 0.10323E-04 100.00 0.0 100.00 0.23387E-02 7.87  
2 0.10948E-02 7.83 -0.10845E-02 16.23 0.25811E-02 7.13  
3 -0.14106E-02 5.08 0.14210E-02 3.31 0.33906E-02 5.43  
4 0.17134E-08 100.00 0.10321E-04 100.00 -0.54569E-02 3.37  
CRITICAL SPEED = 0.22073E+03  
MAXIMUM DEFLECTION = 0.10422E-01

LOAD CONDITION 2  
PLY EPL S.F. EPT S.F. EPLT S.F.  
1 0.12903E-05 100.00 0.0 100.00 0.34301E-02 5.36  
2 0.15935E-02 5.38 -0.15922E-02 11.05 0.37946E-02 4.03  
3 -0.20812E-02 4.12 0.20825E-02 2.26 0.43640E-02 3.71  
4 0.25129E-08 100.00 0.12878E-05 100.00 -0.83035E-02 2.30  
CRITICAL SPEED = 0.22102E+03  
MAXIMUM DEFLECTION = 0.10050E-01  
WEIGHT = 0.43912E+04  
VOLUME = 0.78414E+05

Figure 15. Initial Design for Example 6.





# COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4  
NUMBER OF MATERIAL TYPES = 1  
NUMBER OF LOAD CONDITIONS = 2  
ECCENTRICITY = 0.10000E+00

DIMENSIONS  
PLY THICKNESS PERCENT DIAMETER THETA  
INSIDE 0.10611E+02  
1 0.12351E+00 3.85 0.10858E+02 0.0  
2 0.15427E+01 48.08 0.13943E+02 0.42241E+02  
3 0.15427E+01 48.08 0.17029E+02 -0.42241E+02  
4 0.23842E-06 0.00 0.17029E+02 0.90000E+02

STIFFNESS  
AE = 0.11016E+10  
EI = 0.27109E+11  
GJ = 0.37780E+11

LOADS:  
L.C. T M F H.P. RPM  
1 0.14324E+08 0.0 0.40000E+05 0.25000E+05 0.11000E+03  
2 0.21008E+08 0.0 0.50000E+04 0.10000E+05 0.30000E+02

LOAD CONDITION 1  
PLY FPL S.F. EPT S.F. EPLT S.F.  
1 0.36312E-04 100.00 0.0 100.00 0.20583E-02 8.94  
2 0.13354E-02 6.42 -0.12991E-02 13.55 0.21805E-03 84.34  
3 -0.15867E-02 5.40 0.16230E-02 2.90 0.34659E-03 53.09  
4 0.10136E-08 100.00 0.36311E-04 100.00 -0.32282E-02 5.70  
CRITICAL SPEED = 0.19758E+03  
MAXIMUM DEFLECTION = 0.13017E-01

LOAD CONDITION 2  
PLY FPL S.F. EPT S.F. EPLT S.F.  
1 0.45389E-05 100.00 0.0 100.00 0.30139E-02 6.09  
2 0.19319E-02 4.44 -0.19274E-02 9.13 0.36831E-03 49.46  
3 -0.23529E-02 7.64 0.23584E-02 2.00 0.45984E-03 40.01  
4 0.14866E-08 100.00 0.45375E-05 100.00 -0.47347E-02 3.89  
CRITICAL SPEED = 0.19833E+03  
MAXIMUM DEFLECTION = 0.12482E-01  
WEIGHT = 0.18725E+04  
VOLUME = 0.33438E+05

Figure 16. Optimum Design for Example 6.



## V. CONCLUSIONS AND RECOMMENDATIONS

With all other parameters held constant, an increase in speed drives the design to a large inside diameter while reducing the thickness considerably. This trend is so strong that a reasonable lower bound on the thickness must be given to prevent numerical ill-conditioning.

The introduction of eccentricity lets the optimization produce larger values for the design variables.

The effect of axial load is reflected on the deflection. If very small deflections are allowed, the axial load effects are reduced.

The versatility of the program should not be lost in the emphasis on composite shaft applications. The objective function need not be confined to the weight. If the outer dimension is critical to the design, one may set a specific value on the diameter and let all other parameters vary during optimization.

In summary, numerical optimization provides an efficient and effective way of creating composite shaft designs.

The study has shown the feasibility of using numerical optimization techniques in the design of composite shafts within the limitations imposed in the analysis. Further studies on the same design field may be pursued by eliminating some of these limitations. For example, the unfeasible region defined by the speed constraint extends only to some discrete distance and a supercritical feasible design area does exist. It will



be interesting to develop optimization techniques to accomodate this disjoint design field and to investigate the peculiarities this will introduce to the anlaysis.

Another investigation may be concerned with shafts with variable thickness and diameter along its length. The inclusion of radial stresses in the calculations, the use of finite element methods in three dimensional stress analysis of the problem, and vibration considerations are but some of the aspects of research that may be useful in future anlaysis of composite materials.



## APPENDIX A

### COMPUTER CODE USAGE

The FORTRAN program used in this investigation is described here. Subroutine ANALIZ provides the basic analysis used in the optimization. It reads the initial design descriptions and calculates the values of the objective function, constraints, and all other parameters necessary to solve the analysis problem. COPES/CONMIN updates the design to minimize/maximize the objective function, iterating until no further improvement in the objective function is possible without violating one of the constraints. This process is shown in the flow diagram, Figure A-1.

The global catalog given in Table 3 lists the locations, FORTRAN name, mathematical symbol and description of the parameters used in the optimization. These parameters are contained in the labeled COMMON block, GLOBCM, which is accessed by COPES for optimization.

There are seven data groups to describe the initial design. The first data group contains the title. One card is used here. The second data group contains the number of plies, NPLY, the number of different materials, NMT, and the number of load conditions, NLC. One card is used, here. The third data group contains the initial inside diameter, shaft length, magnitude of mass imbalance, fraction of shaft radius at which the mass imbalance acts, and the acceleration of gravity. One card is





used here. The fourth data group describes the thicknesses and the orientations of plies and material type used in the analysis. NPLY cards are used here. The fifth data group describes the ply longitudinal modulus, EL, ply transverse modulus, ET, ply shear modulus, GLT, ply major Poisson's ratio, PRLT, and the specific weight, RHO. The ply transverse Poisson's ratio, PRTL, is calculated internally. NMT cards are used here. The sixth datagroup specifies the ply longitudinal compressive strain limit, EPLC, ply longitudinal tensile strain limit, EPLT, ply transverse compressive strain limit, EPTC, ply transverse tensile strain limit, EPTT, and ply maximum shear strain limit, GMLT. The compressive strain limits are negative numbers, the tensile and the maximum shear strain limits are positive numbers. NMT cards are used here. The seventh data group describes the torque, moment, axial load, horsepower, and shaft speed in RPM for each loading condition. NLC cards are used here.

The horsepower, torque, and shaft speed in RPM are interdependent values. If two of them are known, the third one is calculated internally. The following table summarizes the required information for Subroutine ANALIZ.



Table 3: Input Data

Data Group	Information	Format
1	Title	20A4
2	NPLY, NMT, NLC	8I10
3	DIAMI, AL, EO, EC, GRAV	8F10.2
4	TN(I), THND(I), MTYP(I)	2F10.2,I10
5	EL, ET, GLT, PRLT, RHO	8F10.2
6	EPLC, EPLT, EPTC, EPTT, GMLT	8F10.2
7	T, M, F, HP, RPM	8F10.2



SUBROUTINE ANALIZ (ICALC)

Flow Diagram

ICALC = 1

Call Subroutine SHFT06  
Read Initial Dimensions  
Read Material Properties  
Read Design Conditions  
Print all Input

ICALC = 2

Calculate Geometric Parameters  
Call Subroutine SHFT07  
Calculate Objective Function  
Call Subroutine SHFT03  
Calculate Shaft Section Properties  
Call Subroutine SHFT09  
Calculate All Constraints  
Call Subroutine SHFT10  
Calculate Critical Frequency  
Call Subroutine SHFT11  
Calculate Maximum Deflection

ICALC = 3

Call Subroutine SHFT08  
Print Analysis Results



# OPTIMIZATION FLOW DIAGRAM

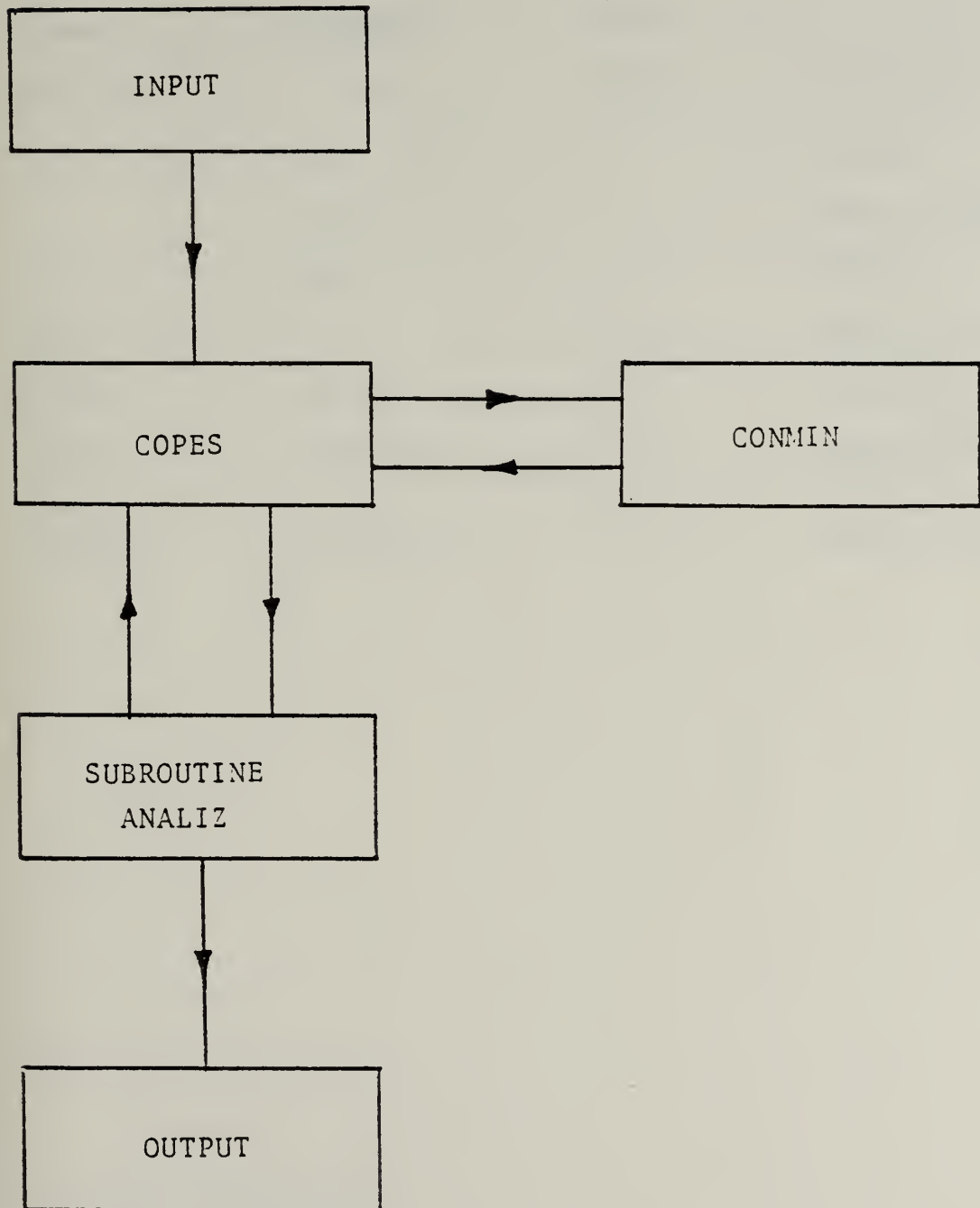


Figure A-1





Table 4: Global Catalog of Parameters

Global	FORTTRAN	MATH	
Location	Name	Symbol	Definition
1	VOL	V	Volume
2	DIAMI	d	Inner Diameter
3	WGHT	w	Weight
4-10			Dummy Storage
11-30	TN(20)	t	Thickness
31-50	THND(20)	$\theta$	Orientation (Degree)
51-551	G(500)	G	Constraints



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